Linear Algebra I

Backpaper Examination

Instructions: All questions carry ten marks. Vector spaces are assumed to be finite dimensional.

1. Find, with justification, the reduced row echelon form of the matrix

$$\begin{bmatrix} 0 & 1 & 2 & 7 \\ 5 & 3 & 1 & 4 \\ 3 & 2 & 6 & 1 \\ 2 & 1 & 4 & 9 \end{bmatrix}$$

2. Without using the concept of determinant, show that the product of two square matrices A, B is invertible if and only if each of them is invertible.

3. Let $\mathcal{B} = ((1,7,5),(0,0,1),(4,5,6))$ denote a basis of $\mathbb{R}^3$. Compute the coordinates of the three standard basis vectors e_1, e_2, e_3 with respect to $\mathcal{B}$.

4. Show that the set of real symmetric matrices of size n is a vector space over the field of real numbers. Find a basis of this vector space.

5. Let L be a linearly independent subset of a vector space V over a field F. Prove that L is contained in a maximal lineraly independent subset of V.

6. State and prove the dimension formula for a linear transformation between two vector spaces.

7. Let V be a finite dimensional vector space over the field of real numbers. A linear operator $T : V \to V$ is called a *projection* if $T \circ T = T$. Let K and W be the kernel and image of T. Prove that $V = W + K$ and $W \cap K = \{0\}$.